Explanatory Document for the Quantitative and Symbolic Reasoning SLO's

What is Quantitative and Symbolic Reasoning: A needed clarification on terminology that will be explained in more detail in the document. The term "Quantitative Literacy" has two applications in the Gen. Ed. program: As an outcome to BACR courses, and As a dedicated course. Hence forth Quantitative Literacy will refer to the BACR outcome and Quantitative and Symbolic Reasoning will refer to the dedicated course. The Quantitative and Symbolic Reasoning committee is a subcommittee of the Eastern Washington University (EWU) General Education (Gen Ed) committee. It was given the mandate to construct the student learning outcomes (SLO's) for the new general education quantitative and symbolic reasoning course and to construct a template for use by anyone wishing to submit a course to be considered as a quantitative and symbolic reasoning course. Early on the committee identified three main themes to work from:

1. A quantitative and symbolic reasoning course is not the same as the breadth area quantitative literacy outcome in the EWU general education programme. The main point was that in order for a course to satisfy the quantitative and symbolic reasoning requirement the predominant feature of the course must be quantitative reasoning/literacy as opposed to quantitative literacy being a supplement to the course.
2. The quantitative and symbolic reasoning requirement should be reasonably close to the equivalent requirements at our peer institutions, which we identified as Central Washington University(CWU) and Western Washington University(WWU).
3. We also wanted to define the requirement without proscribing which courses or departments the courses must come from.

Quantitative and Symbolic Reasoning at peer institutions

CWU: At CWU the "quantitative and symbolic reasoning" requirement is a two course requirement.

- Level 1: One of 6 mathematics courses. There is one course at a lower level than at EWU, which is intermediate algebra. The next lowest level is precalculus I.
- Level 2: This consists of three courses; Math 130 (finite mathematics: EWU Math 107), Philosophy 201 (intro to logic), CS 105 (logical basis of computing)

WWU: At WWU the quantitative literacy requirement has three paths

- Path 1: This is a 2 course sequence: one of Math 107, 108 or 112 (equivalent to or related to Math 107 at EWU) and one more course from a selection of courses given by different departments.
- Path 2: This is a 1 course sequence starting at precalculus I (Equivalent to Math 141 at EWU)
- Path 3: A two course sequence designed for elementary education students.

Using these as a template we identified that the common theme of each programme was that they were built on an axiomatic structure which model of deductive logic. The courses that satisfy the requirements at peer institutions have at least one mathematics course and/or a logic based course at a Grade 11 or Grade 12 level. We now look to define what is meant by an axiomatic structure so that it will not be limited to certain disciplines, but will also maintain a common standard with our peer institutions.

Quantitative or Symbolic Reasoning Structures: Mathematicians and logicians study systems of rules. They explore these systems to study the consequences of the rules and decide if the system is a good system or not. A key idea of these systems is that they are brittle. Rules(axioms) within the system cannot be bent, broken, or interpreted without breaking the entire system. In order to be an axiomatic system, the rules must be designed to be followed exactly with no exceptions. Computers are great at following rules exactly, and this gives us a rule of thumb: If you can write the rules for how to do it in a programming language and the computer does it, it's an axiomatic system. If you can't program a computer to do it, then it's probably not axiomatic. If a computer does it, but there's something wrong about how the computer does it, then either you made a mistake in your rules, or what you're trying to do is not axiomatic. The primary focus of any quantitative or symbolic reasoning course should be the construction of and the use of an axiomatic structure while particular applications are a secondary focus. Quantitative and symbolic reasoning structures form the basis of any deductive logic system.

Examples: Quantitative or Symbolic Reasoning systems:

- Algebraic manipulation: Defines rules for manipulating mathematical expressions and equations. When the rules are not followed, the answer is not correct.
- Euclidean Geometry: Defines and studies properties of objects on a perfectly flat surface. When the rules are changed, we study objects in a different universe. For example, the rules for triangles on the surface of a sphere are different than the rules for triangles on a flat surface.
- Formal Logic: Defines symbols and rules for judging whether an argument is valid within the system or not. When these rules are not followed, the argument is not guaranteed to be valid. Note that validity of an argument does not depend on any "real world" interpretation of the words used.
- Programming Language: Rules are interpreted exactly by a computer. When an error is made, the program does not work as intended, or does not work at all.
- Statistical modelling: The rules of probability determine what conditions an experiment needs to meet for a particular statistical test to be valid. When these conditions are not met, the statistical result cannot be interpreted correctly.

Systems that do not represent a Quantitative or Symbolic Reasoning system:

- The Revised Code of Washington: People break laws all the time, and the government does not cease to exist.
- The English Language: English has a lot of style guide rules of grammar, and word formation, but people constantly break these rules and can still be understood. Ex: "This sentence, no verb". Or a sentence can be constructed using the rules of grammer that is semantically nonsense. Ex. Chomsky's famous sentence *"Colorless green ideas sleep furiously"*. This is different than a linguistics or computing science course which may study the formal structure of languages independent of actual languages.
- Biological or social phenomena: We might use equations to model these systems, but the actual biological or social behavior is too complex to describe completely. The equations form an axiomatic system that is used to describe something that is not an axiomatic system.

For a little more focus on what is meant by quantitative or symbolic structure we look at the three logics of science: Induction, Abduction and Deduction.

Induction: This is the logic of making observations and noticing patterns. This is where experimentation occurs.

- The sun rose in the east yesterday and the sun rose in the east today, thus the sun will rise in the east tomorrow.
- Every swan observed in the known world is white, thus all swans are white.
- No fossilized rabbit is found in the same layer as fossilized dinosaurs, thus no rabbit will every found with dinosaur fossils.
- The continent of Africa, if moved, fits nicely into the Gulf of Mexico.
- When the supply of a product increases the price tends to decrease.

Abduction: This is the logic of explaining the evidence and the patterns: The logic of Sherlock Holmes. Occam's razor is a line of reasoning in this logic.

- The sun rises because it orbits the Earth
- All swans are white because Mendelian genetics says it is not possible for them to not be white.
- The theory of evolution explains why no fossilized rabbit can be found with dinosaur fossils.
- Continental Drift explains why the map of Africa fits snuggly into the Gulf of Mexico.
- The supply and demand curves explain the relationship between the price and the supply of a product.

Deduction: This is the logic purely based on axiomatic systems. This is where mathematics lives, where propositional logic lives. It is the logic of making predictions. These structures are independent of "real world" settings, which means they have no "meaning" until they are used in conjunction with induction and abduction.

- If the Earth revolves around the Sun, then Euclidean geometry says there must be parallax observed from the Earth to the stars. Greek technology was not good enough to notice that parallax did exist so they rejected the alternate model proposed by Aristarchus that the Earth revolved around the Sun as it spins and stayed with the explanation that the Sun revolved around the Earth.
- Deductive logic says that no black swans will ever be found. When black swans were found in Australia, the explanation had to be modified or thrown out.
- The theory of evolution deductively predicts that the first ancestors of land animals should be found in geological layers that are about 375 million years old. This was inductively confirmed when a scientist choose to look specifically in layers that were that old and they found the transitional fossil Tiktaalik.
- Continental drift predicts that land forms all over the world should match up with other landforms in far distant lands and that animal species should also match up. Hence the Western coastline of Africa matches the coastline of the Gulf of Mexico and South America.
- Given particular supply and demand curves predictions can be made about the results of a change in any one of the curves.

Quantitative or Symbolic Reasoning at EWU: Science is built on the foundation of three logical structures: Induction, Abduction and Deduction. Quantitative or Symbolic Reasoning courses concern themselves predominantly with models of deductive logic called axiomatic structures.

As part of the general education programme the quantitative requirement at Eastern Washington University comes in two forms:

- *BACR outcome: Quantitative Literacy* These courses will predominantly build up a concept using the inductive or abductive logic structures and satisfy the quantitative literacy component with some deductive applications to solve particular problems that arise.

  For example, a BACR course on climate may spend most of its time analyzing the inductive and abductive constructs surrounding climate on Earth. It is likely though that such a course will apply some mathematical formulas to calculate basic ideas like the temperature of the Earth in the absence of an atmosphere.

- *Quantitative or Symbolic Reasoning course:* Here the focus of the course is on one or more axiomatic systems: A purely deductive structure. The course will build on these structures to form a given foundation that can be applied to particular problems. Note that the structure created will have no meaning in "the real world". The given problems in such a course may arise in inductive or abductive settings but the structure itself does not.

  For example, In the mathematical reasoning course MATH 107, The section on finance builds up from scratch the formulas for calculating the payments on a loan. This usually takes two weeks to work through all the details even though only a few formulas are created. The majority of time will be spent creating the formulas step by step (ıtpayment by payment) until the appropriate formula is constructed. In this case the problems at hand will use the deductively constructed formulas.

  – A course where the priority is creating an abductive structure using inductive observations, even if it then uses logical or mathematical ideas to solve problems, would not qualify as a quantitative course. For example; if a course constructs an abductive model of some pattern observed in the real world and then uses systems of equations to solve problems related to that model it would then satisfy the BACR quantitive literacy, but it would not satisfy the quantitative and symbolic reasoning requirement. This is because it is using the given technique of solving systems of equations but it does not justify why that technique works in general: It is a given that the technique works.

  – A course where the priority is creating a logical/mathematical structure which may then be used to solve particular problems is a quantitative course, provided the axiomatic structure is at about a grade 11/12 level. Frequently, this will use abductive models that were constructed in a preceding course, but not this course. In a pre-calculus class the technique of solving systems of equations is examined in detail and justification is given for what is being represented geometrically and for why the technique will always give a correct answer. It is not given that the technique works, that must be justified.

SLO 1: Frame problems in ways that would enable one to impose mathematical or axiomatic structures to them..

In order to play the game, you need to decide on the rules

Students will have many tools in their tool box and now need to figure out which ones to use on a given problem. These toolboxes will contain a mix of rules from the number systems, geometry, logic etc. Frequently the students need to use the rules from different structures when solving a given problem. To solve the given problem students must choose from amongst all of these rules the ones that are most useful to the problem at hand. In addition students may be asked to construct their own rules for a given situation, using rules already known. Thus the focus is on how did the student assign meaning to the formulas in play. A typical question coud be something along the lines of: "*Explain how you would solve the problem?"*. Just because there is a formula it does not follow that it has "meaning", but assigning meaning to it leads to the reification fallacy.

Examples:

- Geometry has many different forms; Euclidean Geometry, Spherical Geometry, Hyperbolic Geometry or Projective Geometry. An art student can be working in any of these settings depending on the image they are creating. For example, Escher's circle limit prints use the rules of Hyperbolic Geometry while understanding perspective needs the rules of Projective Geometry. The course will have build these geometric structures from scratch for use in later situations. Here the "how did I decide question" could be answered with: " I recognize that I am working with shapes on a sphere, thus I will use Spherical Geometry". The artistic representation of spheres is being imposed onto the formal spherical geometry.
- In Finance, to build up the rules of simple interest, compound interest or savings plans, one needs to work with the basic rules of number and the rules of exponentiation to build the equations from scratch. Once these equations are understood, they in turn will become the rules needed to calculate payments needed for a loan. A question could be something along the lines of "how are you going to solve this question?". An answer could be: "The problem requires loan payment so I chose a formula with *pmnt* in it". Not the best answer, but the point is the formula contains the variable *pmnt* and now the student is imposing meaning onto that variable.
- In order for a student to determine if an argument is valid or not, they need to work with the rules of logic together with the definitions(rules) of argument and valid argument. Determining whether an argument is sound is not part of the axiomatic system as it requires the interpretation of the truth of statements in the world around us and that is not consistent from person to person.

SLO 2: Evaluate the appropriateness of a mathematical or axiomatic structure to a problem.

Are they good rules

In addition to selecting an appropriate mathematical or axiomatic structure for addressing specific problems the students should be able to justify their choice of structure that they will be working with. In any course, students may learn multiple concepts that apply to the material studied in the course. A specific axiom or rule will be appropriate for solving some problems, but another axiom or rule will be needed in other settings. Students are accountable for applying the correct axiom or rule for specific problems. In many instances more than one set of rules can be applied to a single problem and part of the work for the student is choosing the rules that simplify the work needed to solve the given problem.

Example:

- The art student from the previous SLO should now be able to explain why he is using spherical geometry for his work and not Eucliden geometry.
- A student in a formal logic course should be able to identify that a derivation using a system of introduction and elimination rules could be far less cumbersome and time consuming than a truth table may be for demonstrating the validity of an argument.
- In statistics there are many formulas(rules) for calculating the *mean* of a set of data. After deciding on the particular rule for the mean the student will need to explain why this particular choice is appropriate for the given problem.
- In Physics a student may have to decide on whether to apply the rules of Quantum Theory, the Rules of Relativity or a combination of both. Deciding on the appropriate mathematical structure to a given physical system can be the most difficult part of a given problem and the student will need to be able to justify their decision. Deciding on the rules is handled in SLO1, while justifying the rules is an SLO2 issue.
- The rules for calculating interest on savings will depend on whether the interest accrued is simple interest, compound interest or in other situations. A student will need to determine which of the rules for interest is most appropriate to the situation at hand and explain why they chose a particular formula and not another.

SLO 3: Apply the mathematical or axiomatic structure to resolve problems.

Follow the rules

Once the rules have been decided on the student will use these rules to solve problems. They need to avoid the common and uncommon issues that arise in the given situation.

Examples:

- To determine if an argument is a valid argument the student must be able to perform the appropriate logical derivation. In doing so, they would need to avoid pitfalls common to novice practitioners, such as misusing the procedures for introducing conditionals, eliminating disjunctions, and so forth.
- Calculating the payment for a 30 year mortgage involves a quite complicated formula and has many traps and pitfalls. Students need to be able to identify that the calculations that they are doing using a calculating device are correct.
- Once a formula is determined for calculating the standard deviation of a set of data, every student needs to be able to utilize the tools at hand to now calculate a correct answer.

SLO4: Evaluate the reasonableness of a solution to a given problem.

What did I write?

At a base level, this would involve the students ability to evaluate whether a solution makes sense in the context of the mathematical/logical structure or analytic strategy being used, and to recognize when it does not. In a broader sense, this would refer to the students ability to understand the solution in the context of whether it addresses the question adequately and which, for example, might point them to reconsider the appropriateness of the analytic strategy being used.

Examples:

- If a student is calculating a correlation coefficient and obtains an answer of r = 2.3, then they should be able to recognize that this reflects an error given the numerical range of r (from -1.0 to 1.0).
- A student calculates the distance from Miami to Havana using Eratosthenes technique and obtains an answer of 100,000 miles. The student needs to identify that an error has occurred and, if possible, find and fix the error.
- A student should recognize that a loan payment of $2000 per month will likely be too high for a 15 year mortgage of $300,000. Can the student recognize that the loan agency they are working with may not be reputable based on this.
- In a logic class a student should be able to identify and rectify an issue when they use different techniques and if they find that a given statement is both a tautology and not a tautology.

SLO5: Communicate the strategies used at a level suitable for an audience of their peers

Show your work

Students need to be able to present and describe the axiomatic structure they select for a given problem, explain why that axiomatic structure is appropriate, and analyze its implications in a language and depth suitable for their peers. They should do so using the different mathematical and logical tools suitable for the axiomatic structure and the problem under study. This part is a synthesis of all that has come before.

Examples:

- In a geometry class a student who has constructed an Escher Tiling on the Euclidean plain should be able to explain the methods used for the construction in such a way that a peer can then create their own tiling.
- A student who is calculating the payment plan for a 15 year mortgage should be able to describe the assumptions and calculations that are necessary for finding the monthly payment. Amongst the assumptions and calculations needed would be the calculation of the monthly compounded interest rate and what the calculation means.
- A student studying the elastic force that a spring experiences when a mass is hung on it should be able to describe the response of the spring with words in English. They should also be able to explain the use of the equation that describes that behavior (Hookes law).
- After determining whether a given argument is valid or not, a student should be able to explain to a fellow student how they solved the problem, which rules were used and which were not, so that their peer can solve a similar problem independently.

Requirements for submission: The following represents the criteria that needs to be discussed for any course when applying to be considered as a quantitative literacy course,

1. Explain the Quantitative or Symbolic Reasoning structures (axiomatic structures) that form the focus of the course.
2. Explain how the topics to be covered in this course have the Quantitative or Symbolic Reasoning structures as their predominant feature. Provide examples of course topics, indicating how much time in the course will be devoted to each topic.
3. Indicate with examples, how this course's content addresses each of the quantitative or symbolic reasoning Student Learning Outcomes.
4. Provide example assignments, with corresponding rubrics, that could be used to evaluate whether students have successfully achieved each of the Student Learning Outcomes.

Quantitative Reasoning, Student Learning Outcomes.

Students successfully completing this course will be able to

SLO 1: *Impose mathematical or axiomatic structures onto problems.*

SLO 2: *Evaluate the appropriateness of a mathematical or axiomatic structure to a problem.*

SLO 3: *Apply the mathematical or axiomatic structure to resolve problems.*

SLO 4: *Evaluate the reasonableness of a solution to a given problem.*

SLO 5: *Communicate the strategies used at a level suitable for an audience of their peers*