RAMP-A
September 26-27, 2014

Integrating and applying characteristics of the CCSS: focus, coherence, rigor, and standards of mathematical practices.

We’ve come so far!

- We’ve worked to understand the CCSS Algebra and Functions Conceptual Categories.
- We’ve explored and used the Standards for Mathematical Practices while doing math.
- We’ve learned some instructional strategies to support students’ learning of the CCSS with focus and coherence.
- We’ve examined the use of formative assessment and cognitive demand to support our students’ learning.
- We’ve developed lasting relationships and ways of working to support our continued learning.
- We’ve discussed students’ motivation and ways to increase it.

Evaluation Results

- Poster presentation at the national MSP meeting in Washington DC next week.

Goals of the Grant

Deep common understanding of the Algebra 1 content in the CCSS

Increase student motivation, engagement and interest in mathematics

Thoughtful planning, instruction and assessment

Professional reflection, support and growth in partnership with colleagues and community

Think for a moment

- What ways of working and spending your time do you think could most contribute to improving your students' learning of the CCSS?

Respond

- http://today.io/rvNX

Expectations for the year

- Four full days and four half-days of workshops
- 20 hours of Homework outside of workshop hours
  - Individual
  - Meet with PLC and report back to whole group
  - Online materials
- Summer Institute: June 23, 24, and 25
- Fall and spring observations/coaching
- Ongoing communication with administrators and team members

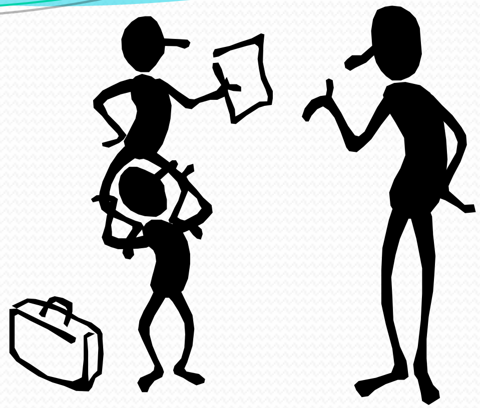
Goals for today and tomorrow

- Drill deeper into teaching the CCSS by:
  - Examining what teaching and learning for coherence, focus, rigor, and SMP look like.
  - Investigating to better understand students' difficulties.
  - Practicing and planning Ten-Minute Talks that support coherent learning.
  - Do some math.
  - Share ideas that support learning the CCSS.
  - Find a resource and plan a way to use it.

Group Processes and Collaboration

- Know our <u>intentions</u> – use to guide actions
- Maintain <u>productive</u> ways of listening, responding, and inquiring
- Know when to <u>help the group</u> refocus or stay on task, and when to integrate (attempt to understand others' viewpoints)
- Know and support the <u>group purposes</u>, processes, and development
- Keep an <u>open mind</u> – wonder about possibilities

CCSS Card Sort



- Critical Friends groups
- Two types of cards: term, definition
- Silent Card Placement
  - Each person, in turn, places one card on the table – either by itself or next to a card or cards that it matches – or moves a card that had been placed earlier.
  - Continue until all cards are placed and all sets have one of each type.
- Group Card Creation
  - Card #1: What will (term) look like from a teacher's perspective?
  - Card #2: What will (term) look like from a student's perspective?

Break

Do you have something that others here could benefit from knowing? Have you attended an event or been involved in a project that you'd like to share? Perhaps a resource, a project at your school, or a state or national effort: TPEP, State Math Fellow, COE, STEM-PD, Dan Meyer, Jo Boaler, etc…

If so, write your name, a very brief description of what you can share, and approximately how much time you will need on a 3-by-5 card and give it to Janet.
We'll organize these for tomorrow.

Understanding students' difficulties with x=x+1

Goal: To better understand students' difficulties in an investigative way.

Episode 1

Two students were working together to solve an equation. They simplified both sides and arrived at

$$x=x+1.$$

Dan: Let's subtract x from both sides.

Jessica: But if so, we get 0=1. Something is wrong. We cannot solve it.

How would you help these students so that they solve the original equation with depth and connections?

Issue: Keeping *x* in the solution process

Teacher:

'0=1' confuses students because they think they are supposed to solve for x, but x disappeared. Students should keep x in the equation. So I will have them stop where x=x+1 and not proceed to 0=1.

What do you think about the teacher's idea?

How would you respond if your students say, "Why should we not move to the next step using the skills we know such as subtracting x from both sides?"?

Issue: Keeping *x* in the solution process

Teacher:

Since x+1 is always 1 more than x, x = x+1 has no solution.

What do you think about the teacher's idea?

How would your students respond to the teacher's idea?

Student: I know x+1 is more than x, but what I had a trouble with is 0=1.

How would you respond to the student?

Issue: Mathematical logic

Teacher: 0=1 is false. Therefore, x=x+1 has no solution.

How would you respond if your student says the following?

Student: I don't understand "Therefore" in your explanation. I know 0=1 is false, but don't know how it leads to no solution of the equation x=x+1.

Teacher: You started with x=x+1 and got 0=1 by subtracting x from both sides. The reason you got 0=1 is because there is something wrong with x=x+1. So, 0=1 tells you something about x=x+1.

What do you think about the teacher's response?

Issue: Mathematical logic

Teacher A:

x=x+1 is false because 0=1 is false. Therefore, x=x+1 has no solution.

What do you think teacher's idea?

Teacher B:

There is nothing wrong with x=x +1 as an equation. Saying "x=x+1 is false" may confuse students.

What do you think about the teacher's concern?

Issue: Making *x* visible

Teacher: Students had difficulty connecting 0=1 and the reason for no solution of x=x+1. What if we make *x* visible by writing $0 \cdot x = 1$ instead of 0=1?

What do you think about the teacher's idea?

Issue: Making *x* visible

Students: How could we get $0 \cdot x = 1$ from $x=x+1$?

I solved it the following way,

$x= x+1$

$x - x = x+1 - x$

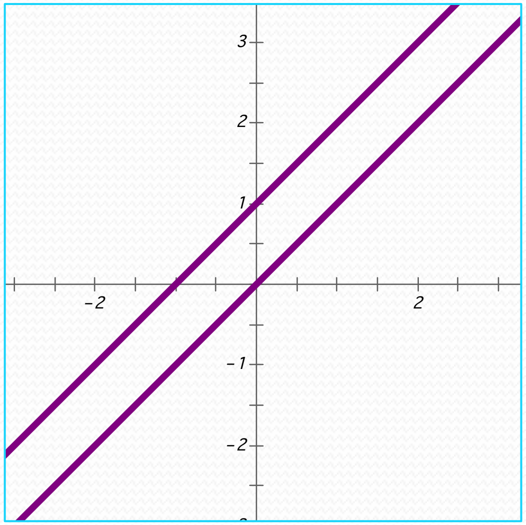
$0 = 1$

I don't get $0 \cdot x = 1$.

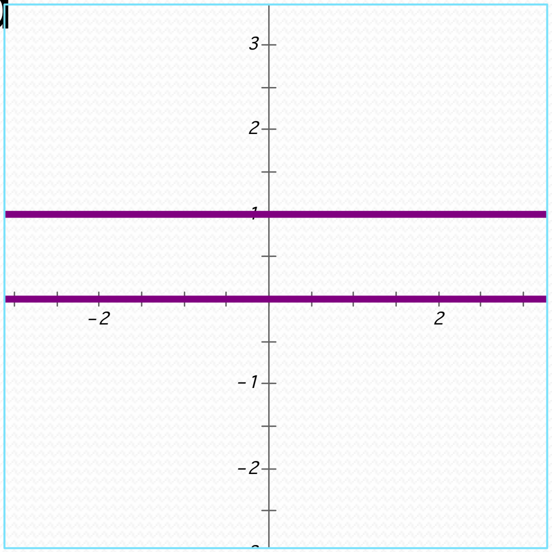
How can we write $0 \cdot x = 1$ as equivalent to $x=x+1$?

Issue: A system of equations

Teacher A: x=x+1 can be solved using a system of equations. It is easy to solve it graphically.



Teacher B: (teacher draws y=x and y=x+1 on the coordinate plane.) The students got stuck with 0=1. How could the graphs help the students see a relationship between 0=1 and no solution of x=x+1?



Discuss the Teacher B’s question.

Episode 2

Two students were working together to solve an equation. They simplified both sides and arrived at x=x+1.

Dan: Let's subtract x from both sides.

Jessica: But if so, then we get 0=1. Something is wrong. We cannot solve it.

Fred: We are supposed to isolate x. I saw Mrs. C square both sides of an equation. We were told we are okay as long as we are doing the same thing on both sides. Let's square both sides of x = x+1. (See his solution on the right.) The solution is – ½.

How would you respond to the Fred's idea?

$$\begin{aligned} x &= x+1 \\ x^2 &= (x+1)^2 \\ x^2 &= x^2+2x+1 \\ -x^2 & \qquad -x^2 \\ 0 &= 2x+1 \\ -2x &= 1 \\ x &= -\frac{1}{2} \end{aligned}$$

Issue:
Developing a habit of checking an answer.

Teacher: In general, students should check their answers after solving a math problem.

How would you respond to Fred if he says, “Is there any particular reason I should check my answer”?

Issue:
Developing a habit of checking your answer.

Teacher: An equation starts with an assumption that there is an unknown making the equation true. We perform all procedures under this assumption. But the assumption may not be true.

What do you think about the teacher's comment?

Issue: Extraneous solution

Teacher: Fred, you solved $x^2 = (x+1)^2$. You didn't solve $x= x+1$.

Fred: I don't agree with you. I did the same thing on both sides of $x=x+1$. So, $x=x+1$ is the equation I solved.

How would you respond to the Fred's response?

Issue: Extraneous solution

Teacher: Fred, you squared both sides of an equation. But squaring does not necessarily give you equivalent equations. Look at what happens when you square both side of x=1. $x^2 = 1^2$. x can be 1 or -1.

Student: How do we know which operations work and which operations don't?

How would you respond to the student's question?

Issue: Extraneous solution

Teacher: Squaring does not necessarily give you equivalent equations.

Student: No, I did not square both sides. I multiplied the same thing on both sides. I got

$x \bullet x = (x+1) \bullet (x+1)$ by multiplying x to the left side and x +1 to the right side of $x=x+1$. x and x+1 are the same.

How would you respond to the student's argument?

Issue: Necessary and Sufficient

Teacher: When we say two equations are equivalent, one has to be necessary and sufficient for the other. This means if one is true, then the other is true, and vice versa. For example, the second to the last line is equivalent to the last line. But the first line is not equivalent to the second one.

$$
\begin{aligned}
x &= x+1 \\
x^2 &= (x+1)^2 \\
x^2 &= x^2+2x+1 \\
-x^2 & \quad\; -x^2 \\
0 &= 2x+1 \\
-2x &= 1 \\
x &= -\frac{1}{2}
\end{aligned}
$$

Student: I don't think so. If the first line is true, then the second one is also true because I multiplied the same thing on both sides of the first one. The other way is also true. If the second line is true, then the first one can be obtained by dividing by the same thing on both sides of the second one.

How would you respond to the student?

Reflect

- How has your understanding of students' sense-making been extended?

Discuss another problem

Students are working on A-CED.2: Create equations in two or more variables to represent relationships between quantities. You notice several students give similar answers to the following:

> Define variables and write an equation to represent: In the school gym there are four times as many basketballs as volleyballs.

As you walk around the room, the three most common answers are:

$$4b = 1v$$

$$b = 4v$$

$$b + v = T$$

Why are students having a hard time?

What activities could you ask them to engage in to better understand?

Becky: SBAC (and Lunch)

- Teachers grouped with Critical Friends

What is the role of *meanings* in learning math?

‘Meanings’ in math class

- As early as 1st grade, the CCSS allude to *meanings:* students “develop meaning for the operations of addition and subtraction”
- Think alone for a couple of minutes about what you are currently teaching and what *meanings* you want students to develop.
- Discuss these in your group, and find standards in the CCSS to help you refine and support your ideas.
- Examine the SMP to identify how students who are proficient use meanings.

Some *meanings* in the SMP: Mathematically proficient students

- SMP 1: explain to themselves the meaning of a problem…conjecture about the meaning of a solution
- SMP 2: contextualize and decontextualize; create a coherent representation attending to the meaning of quantities.
- SMP 6: state the meaning of the symbols they choose…

How do we teach meaningfully?

- Students take time to develop deep understanding of mathematical meaning.
- We need to work with their **current meanings** to help them achieve more refined or advanced meanings.
- Therefore, we need to explicitly identify and articulate both their current meanings and the meanings we intend for them to acquire, which are our instructional goals.

Number Talk

- http://www.insidemathematics.org/classroom-videos/number-talks/7th-grade-math-whats-the-savings/number-talk

Ten-Minute Talk: Meanings

- Role-play
- What does 60 mph mean to you?

Discuss roles of teacher and students:

- Why did I have you do it alone first?
- Why did I have you discuss with an elbow partner, and what is the effect of this?
- Why did I record your answers without comment, or with only clarifying questions?
- What seemed to be my general demeanor?
- What SMP did we use and how did we use them?

Reflect on the Meanings

- What *clarifying* questions would you have asked that I didn't ask?
- Choose one response that is unclear to you and consider what you would ask the student to clarify.

'The odometer tells me that I am going 60 mph."

Next steps:

What can you infer about students' current meanings?

What do you still wonder about students' meanings for 60 mph?

Devise a question, short problem, or prompt that you could use to further understand students' current meanings of 60 mph.

CCSS related to this meaning

- 6RP.1 (ratio as a relationship between two quantities), 2(unit rate a/b associated with ratio a:b), and 3(solve unit rate problems using different representations).
- 7RP.1 (compute unit rates associated with ratios of fractions) and 2 (recognize and represent proportional relationships)
- 8EE.5 (Graph proportional relationships, interpreting the unit rate as the slope of the graph) and 7 (solve linear equations in one variable)
- N-Q.1 (use units as a way to understand problems)
- F-BF1 (write a function that describes a relationship between two quantities)
- F-LE.1a (prove that linear functions grow by equal differences over equal intervals)
- SMP 2 Reason abstractly and quantitatively
- SMP 6 Attend to precision

Practice

- In your groups, plan and carry out a Ten-Minute Meanings Talk using “What is the meaning of a slope of 2?” Poster this!

1. Choose a ‘teacher’
2. Together, examine the meanings and explain what you know and what you still wonder. What prompts would you use to explore student thinking?
3. What new prompt would you use for the next talk?

Break!!

Ten-Minute Talks: Structure

- Talk in your groups:
  - Come up with what you think is meant by ‘structure sense’ and what some of its characteristics might be. Include examples.
  - Why might structure sense be important?

Structure sense (Hoch & Drefus, 2004, p. 51)

- “Structure sense, as it applies to high school algebra, can be described as a collection of abilities. These abilities include the ability to:
  - see an algebraic expression or sentence as an entity,
  - recognize an algebraic expression or sentence as a previously met structure,
  - divide an entity into sub-structures,
  - recognize mutual connections between structures,
  - recognize which manipulations it is possible to perform, and
  - recognize which manipulations it is useful to perform”

Elements of structure sense include:

- *symbol sense* (meaningful use of variables, grouping symbols, and equal signs),
- *property sense* inherited/extended from arithmetic or number sense (ie. associative, commutative, distributive, additive, multiplicative),
- *operation sense* (meanings of and relationships between operations).

- Structure sense is important to support the idea of equivalence of expressions and ability to reason with equations and inequalities. It is also important for students to be able to distinguish between objects they are working with.

"The ability to see an algebraic expression or sentence as an entity necessitates stopping to look at the equation before automatically applying algebraic transformations."

Discuss in your groups

- What difficulties have you noticed with students' structure sense?
- Consider the topics you are currently teaching and identify opportunities to develop structure sense.
- Look for references to structure in the SMP: in what ways do students proficient in the mathematical practices use structure?

SMP and Structure

- SMP1: “transform algebraic expressions…to get information they need;…draw diagrams of important features and relationships”
- SMP 2: “represent it symbolically and manipulate the representing symbols as if they have a life of their own…; knowing and flexibly using different properties of operations and objects”
- SMP 4: “analyze relationships mathematically to draw conclusions”
- SMP 7: “look closely to discern structure; see complicated things such as some algebraic expressions, as single objects or as being composed of several objects…”
- SMP 8: “notice when calculations are repeated, look for general methods and shortcuts, … abstract the equation… notice the way terms cancel”

Ten-Minute Talk: Structure

- Role-play
- For what values of x is $(x-3)^2 - 4$ positive?

Discuss

- **Reflect on students' responses**: What clarifying questions would you have asked that I didn't ask? Choose one response that is unclear to you and consider what you would ask the student to clarify.
- My next week's question is:

  Are $(x-3)^2$ and x^2-3^2 the same?
- What do you still wonder about students' sense of structure? What can you infer about students' current structure sense?

Reflection

- What effect might a Ten-Minute Talk have on students with low motivation, low achievement, or low prior knowledge?
- How would a Ten-Minute Talk play out in your classroom?
  - What might go wrong?
  - What might students like or do well at?

Saturday: 8-12, Riverpoint Campus, SAC 241

Saturday Morning

- Welcome back!

Wireless:
Username: RAMP-A
Password: R8V6U4f3

Scott: Norms for Doing Math

- Allow quiet think time, begin talking when everyone is ready.
- Offer help, not solutions when we ask.
- Ask for help when you need it.
- It's math, have fun!
- Stay on task and persist in problem solving.
- Stay open minded to alternative ways of thinking.
- Take responsibility for making contributions to your group.
- No one is done until everyone is done.

Create!

- Make up an arithmetic sequence of six numbers: a_0, a_1, a_2, a_3, a_4, and a_5. Then write and solve the system of equations (feel free to use technology to help you solve these).

$$a_0 x + a_1 y = a_2$$
$$a_3 x + a_4 y = a_5$$

Compare & Conjecture

Compare your answer to others who used a different sequence. Come up with conjectures as to both what is happening and why.

Go Deeper, or Explore Wider

Based on what you just saw, what other explorations would you like to do? Would you like to explore *why* more, or alter the conditions and explore different sequences?

Report out

- What did you explore?
- What did you see?
- What are you still wondering?

On the back

- How could this task help students deepen their understanding of arithmetic sequences and systems of linear equations?

Break:

Teacher Sharing

Exploring Resources

- Choose a new resource or new part of your school "text" (e.g., Engage NY, Math Vision Project) to explore
- What teaching moves might you use with this resource to support students' learning with focus and coherence?
- What meaning(s) should students develop when this resource is used?
- What structure(s) will you help students notice as you use this resource?

Your Tasks due at next workshop

- **Individually**: Plan, use, and reflect on one Ten-Minute Talk per week. Bring notes from ***one*** Ten-Minute Talk to the next workshop that include the prompt, student responses, your wonderings and ideas about ways to respond, and your next prompt.
- **PLC**: Meet at least once (weekly is better) to discuss your Ten-Minute Talks and get your colleagues' ideas about what students could have been thinking and different ways to respond.

Evaluations: Thank you!

- Your evaluations are important to us. They are not just for clock hours, they guide and inform our work.

- Please take time to give thoughtful and complete responses. You may identify yourself or may remain anonymous.